Identification Of Flexible Robot Arm System Using Extended Volterra Series By Kautz Orthogonal Functions

Mahmood Ghanbari, Marziye Abbasi

***Abstract*— This paper presents identification of a non-linear system for robot flexible arm as a black-box using Volterra series model. In most of the methods using Volterra series, too many number of parameters are estimated which is in contrast with the simplification principle in system identification. In this paper, for system identification of robot arm, kernels of Volterra series are developed using orthogonal functions that would reduce the number of parameters and consequently decreasing the identification time and increasing the accuracy and speed of convergence. At last, results of system identification of flexible robot arm based on expansion of Volterra series kernels through kautz orthogonal function are investigated with the results of the other method in literatures.**

***Index Terms*— Volterra Series, kautz Orthogonal Functions, non-linear System Identification, Optimization, flexible robot arm**

I. INTRODUCTION

Despite numerous methods for identification of linear systems, there are a few methods for non-linear systems (Dalirrooy Fard and Karrari ,2005). One of the most comprehensive methods for modelling non-linear systems is Volterra Series. Generally speaking, as modeling with Volterra series would not interfere with the hidden state variables of the system, the possibility of specifying the dynamic behavior of the system is provided without measuring the state variables (Minu and Jessy John ,2012). Volterra Series produces a model for non-linear behaviour of the system like Taylor series, but in Taylor series output is highly dependent to the input in a certain instant Whereas In Volterra series, output is dependent to the input in all the instants (S.Parker et al.,2001). The main advantage of the Volterra series lies in its totality, so many non-linear systems can be modeled by it (Mahmoodi, et al.,2007). This series has high number of parameters; therefore it is named a non-parametric model. Series and theory of Volterra were first introduced by Vito Volterra in 1887 (Volterra ,1887; Volterra ,1958). After that, this series was widely used in identifying the non-linear systems. where a sample application of this series is presented in (Alper ,1965). Literatures about this issue are numerous and most of them try to identify a certain system in a certain application or presenting a new method for calculating the kernels of this series from which ref (Meenavathi and Rajesh,2007) is a good example that uses Volterra series to remove Gaussian noise. In (Dalirrooy Fard and Karrari ,2005), Volterra series is used for modeling and identification of the synchronous generator. Ref (George-Othon et al., 1999) presents the specification of Volterra series kernel with multiple methods. Ref (Dodd and Harrison, 2002 ; Yufeng Wan et al., 2003) use Volterra series for estimation of high rank kernels through reproduced Hilbert space.

In this paper in section 2, Volterra series is described and in section 3, the method for calculation of kernels or factors of Volterra series are presented. Laguerre orthogonal function is described in section 3-1 and expansion of kernel with orthogonal function is presented in section 3-2. Flexible robot arm is illustrated in section 4 and section 5 presents the results of simulation.

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II. VOLTERRA SERIES

Volterra series is generalized idea on convolution in linear system. The relation between input and output in a linear and memory less system is expressed as follow:

$$y(t) = h.u(t) \tag{1}$$

in which output is only dependent to the input in the instant t. In a linear, casual, time-invariant, with memory the relation between input and output is as follow (Doyle et al.,2001):

$$y_1(n) = \sum_{i=0}^{\infty} h_1(m_i).u(n-m_i) \tag{2}$$

The relation between input and output in a non-linear and memory less system of rank two is expressed as follow:

$$y_2(t) = h_2.u^2(t)` \tag{3}$$

The relation between input and output in a linear and with memory system of rank two is expressed as follow:

$$\begin{aligned} y_2(n) &= u(n).u(n) + u(n).u(n-1) + ... \\ &+ u(n).u(0) + u(n-1).u(n-1) + \\ &u(n-1).u(n-2) + ... + u(n-1).u(0) + ... \\ &+ u(0).u(0) \end{aligned} \tag{4}$$

Where it can be written

$$\begin{aligned} y_2(n) &= \sum_{j=0}^{\infty} u(n).u(n-j) + \sum_{i=0}^{\infty} u(n-1).u(n-i) + ... \\ &= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} u(n-i).u(n-j) \end{aligned} \tag{5}$$

Now we can add a proper weight to the series of equation (5) without disturbing the whole problem:

$$y_2(n)=\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}h_2(i,j).u(n-i).u(n-j) \qquad (6)$$

For Volterra series of non-linear system with rank n:

$$y=y_0+y_1+y_2+...+y_n \qquad (7)$$

where y_i is calculated like equation (2) and (5) and y_0 is a constant value (e.g. signal average). Now, substituting y_i in (7) gives:

$$y(n)=y_0+\sum_{m_1=0}^{\infty}h_1(m_1)u(n-m_1)+ \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}h_2(m_1,m_2).u(n-m_1).u(n-m_2)+... \qquad (8)$$

h_n is rank n kernel of Volterra. In fact $h_1(t)$ shows impulse function of linear system and $h_n(t)$ is impulse function of a n-dimensional non-linear system. Equation (8) has too many unknowns that calculation of it is time-consuming. For solving this problem we have to consider that there may not be link between output and former inputs. Besides, series can be shown with kernels of rank 1 to L by approximation. From these points, Volterra series is changed as follow:

$$y(n)=y_0+\sum_{m_1=0}^{M-1}h_1(m_1)u(n-m_1)+ \sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1}h_2(m_1,m_2).u(n-m_1).u(n-m_2) \qquad (9)$$

M (memory of system) and L (rank of system) are almost chosen by trial and error method.

III. . CALCULATING THE KERNELS OF VOLTERRA SERIES

Calculation of Volterra series factors is the main issue in solving the equation and identifying the systems using this series. Identifying non-linear systems by Volterra series is in fact identifying the kernel of such series. There are many methods for this purpose such as recursive algorithm (Doyle et al.,2001), Gaussian input (Doyle et al.,2001), gradian-based search (Suleiman and Monin ,2008) , cross-correlation method (Lee and Schetzen ,1965), method of reproducing kernel Hilbert space (Dodd and Harrison, 2002 ; Yufeng Wan et al., 2003) and etc. In addition, the factors of Volterra series can be obtained directly by expanding the orthogonal functions that leads to less complexity and more accuracy. The main importance of such method is that if we rewrite kernels of Volterra series as an expansion of orthogonal function, series is changed into a linear regression equation (Moodi ,2008) In this paper, number of kernels is also considered symmetrical:

$$h_n^{sym}(k_1,k_2,...,k_n)=\tfrac{1}{n!}\sum h_n(k_1,k_2,...,k_n) \qquad (10)$$

Due to complexity of issues for identifying the kernel of Volterra series and writing limitation, we only illustrate the method for identifying kernel by Laguerre orthogonal function.

IV. KAUTZ ORTHOGONAL FUNCTIONS

Orthogonal Katz function

The two- parameter Katz function is defined as follow:

$$\emptyset_{2n}(z)=\frac{\sqrt{(1-C^2)(1-b^2)z}}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)+z+1}{z^2+b(c-1)z-c}\right]^{n-1} \qquad (11)$$

$$\emptyset_{2n-1}(z)=\frac{\sqrt{(1-C^2)z(z-b)}}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)+z+1}{z^2+b(c-1)z-c}\right]^{n-1} \qquad (12)$$

In which a, b are real value constants related to the poles of Katz function.

$$b=(\beta+\beta)(1+\beta\beta) \qquad (13)$$

$$c=-\beta\beta \qquad (14)$$

Simultaneous and optimal choice of c and b is being investigated. So, to obtain the best choice, these parameters considered constant by applying a certain criteria.

V. EXPANSION OF KERNELS WITH ORTHOGONAL FUNCTIONS

As mentioned, expansion of orthogonal basic functions is an estimation method that reduces the number of parameters and consequently reduces the complexity and increases the accuracy and also changes the series to a linear regression. Now factors are calculated as below(Moodi and Bustan ,2010):

$$h_1(m_1)=\sum_{i_1=1}^{p}\alpha_{i_1}\phi_{i_1}(m_1)$$
$$h_2(m_1,m_2)=\sum_{i_1=1}^{p}\sum_{i_2=1}^{p}\alpha_{i_1i_2}\phi_{i_1}(m_1)\phi_{i_2}(m_2) \qquad (17)$$

Where $\Phi_k(n)$ is the kth orthogonal function at point n. so, the relation between input and output in a Volterra series of rank two is as follow:

$$\hat{Y}(n)=Y_0+\sum_{i_1=1}^{p}\alpha_{i_1}S_{i_1}(n)+\sum_{i_1=1}^{p}\sum_{i_2=1}^{p}\alpha_{i_1i_2}S_{i_1}(n)S_{i_2}(n) \qquad (18)$$

In this equation factors are symmetrical ($\alpha_{i_1i_2}=\alpha_{i_2i_1}$), therefore:

$$S_k(n)=\sum_{m_1=0}^{M-1}\phi_k(m_1)u(n-m_1) \qquad (19)$$

First, M and L and P are chosen. Although the actual values of these parameters are too high, if values with lesser degree have an agreeable error, there is no need to increase the values. Finally the equation (18) is written as:

$$y=U\theta+e \qquad (20)$$

where y is output vector, θ vector of factors and e noise or model error, U and known matrix of S values. Now different identification methods are used in a way that modeling error $\|y-\hat{y}\|^2$ is minimized. One of such methods is linear least square that best parameters are calculated as follow:

$$\hat{\theta}_{LS}=(U^TU)^{-1}U^Ty \qquad (21)$$

The above method has some problems and that is inversing from U^TU. For example maybe det(U^T U) is not zero but be near zero. In this case, matrix is called improper and the reason is the dependency between rows and columns of U^TU that identification would be with less accuracy and high error. There are some reasons for this (kararri,2009):

I. Input is not exciting enough and not able to excite all the modes of the system.
II. The considered rank of model is more than actual rank of system.
III. System is identified in closed-loop operation.

For compensating this problem in least square method, inversing U^TU is not allowed and other techniques are employed. In this paper SVD is chosen for this purpose. Besides, equation (22) is a criterion for feasibility assessment of model under the name of forecasted error (Barve ,2004):

$$e^p = \frac{1}{p}\sum_{i=1}^{p}\sqrt{\frac{\sum_{j=1}^{N-1}(y_i(j)-\hat{y}_i(j))^2}{\sum_{j=0}^{N-1}(y_i(j))^2}} \qquad (22)$$

where P is number of outputs, N number of used data and $y_i(j)$ actual output of i at instant j and $\hat{y}_i(j)$ output of the model of i at instant j.

VI. FLEXIBLE ROBOT ARM

Robot arm is a system with high applicability in industrial systems which are shown in Figure 1 (Ferreira and Serra ,2012).Flexibility of the robot arm is in two types:

- Flexibility due to non-linear variations in Drive system.
- Flexibility due to straining the robot arm.

Data of input and output of the system are categorized into two groups, one group for training that the model is extracted with and the other one for experimenting the model.

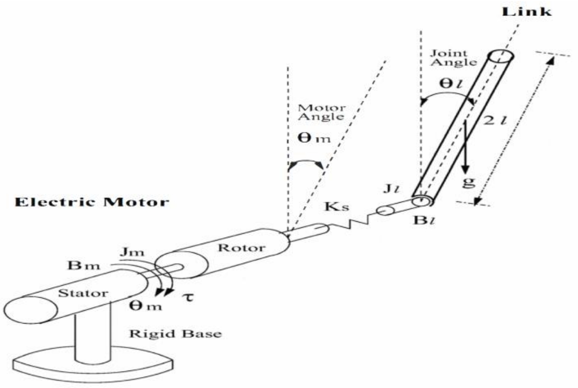

Figure1. flexible robot arm

VII. SIMULATION RESULTS

1. Simulation result

To identify the flexible robot arm to the structure of Volterra series model developed by Katz function will be considered according to Equation 16. First, we need to do optimal estimation of Katz orthogonal function based on the real measured input and output available in the paper of ALEX. According to the model structure, input and output data, the expansion coefficients of the model structure is obtained in IS way.

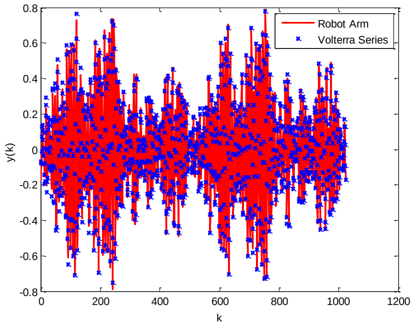

Figure 2. Output waveform of actual system and detected output

To validate and display the success of identification made, the criterion of Mean Squared Error (MSE) and the Normalized Root Mean Square error (NRMS) are calculated as follow:

$$MSE = \frac{1}{N}\sum_{k=1}^{N} e(k)^2 = 5.0891\times 10^{-04}$$

$$NRMS = \sqrt{\frac{\sum_{k=1}^{N}(y(k)-\hat{y}(k))^2}{\sum_{k=1}^{N} y(k)^2}} = 0.0820 \qquad 21$$

VIII. CONCLUSION

In this paper, a comprehensive model based on Volterra series was presented for identification of non-linear system of flexible robot arm. For reducing the complexity and increasing the accuracy of the model, kernels of Volterra series were rewritten as an expansion of orthogonal functions and optimal values of orthogonal functions pole and number of expansion. For preventing inversing problems and impropriety of regression matrix. were compared and it was shown that identification with developed Volterra series by kautz function has less error and more accuracy comparing to other method. In addition, there are other methods of optimization for better adjustment of orthogonal functions which can improve the identification process. This is hoped to be done in future works.

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